MATH GIRLS

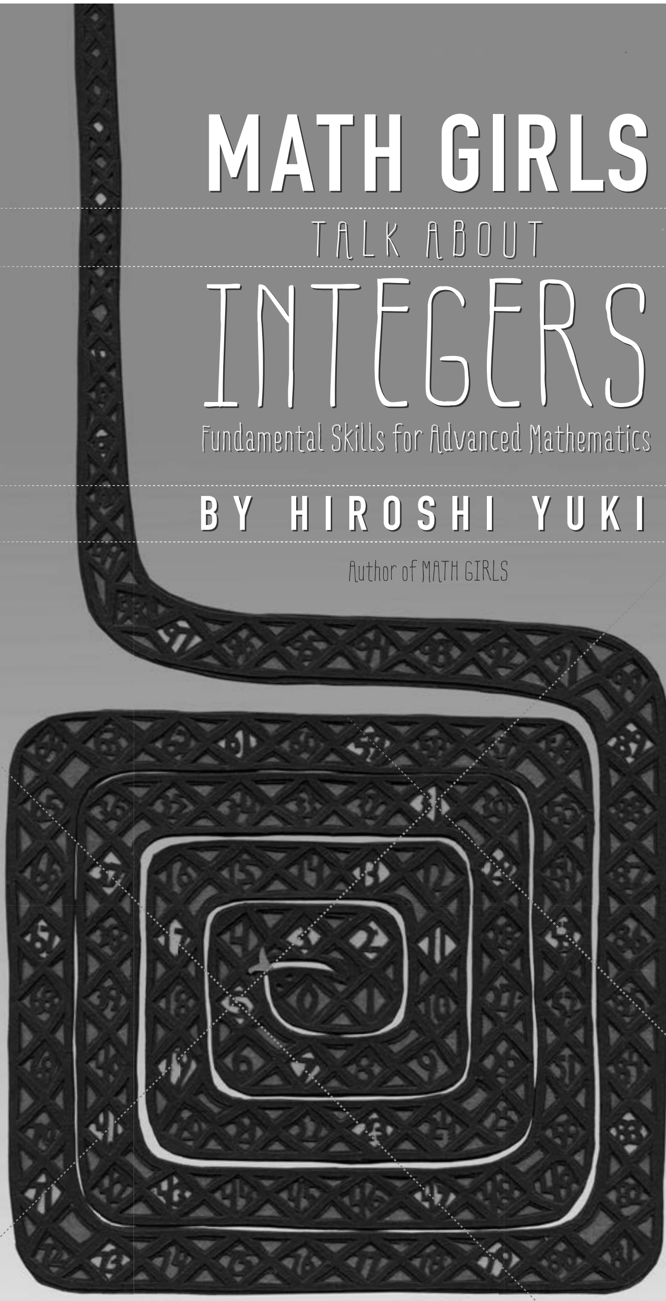
TALK ABOUT

INTEGERS

Fundamental Skills for Advanced Mathematics

BY HIROSHI YUKI

Author of MATH GIRLS



TRANSLATED BY TONY GONZALEZ

MATH GIRLS TALK ABOUT THE INTEGERS

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Math Girls Talk About
The Integers

Contents

Chapter 1

Checking for Multiples

> "You don't have to understand how rules work to use them . . . "

1.1 Multiples of Three

Yuri "Hey, cuz! I have a quiz for you."

Me "I love how you keep thinking you're going to trip me up some day."

Yuri "That day might be today. Tell me this: is 123,456,789 a multiple of 3?"

Problem

Is 123,456,789 a multiple of 3?

My cousin Yuri was in eighth grade, and we'd pretty much grown up together. She lived just up the street, and often came over to read books and work on math problems.

Me "I guess you just made up a big number by stringing together the digits 1 through 9?"

Yuri "That's beside the point. Is it a multiple of 3?"

Me "Yes."

Yuri "Couldn't you at least *pretend* you had to think hard to get the answer?"

Answer

Yes, 123,456,789 is a multiple of 3.

Me "For such an easy question? No. All you have to do is add all the digits in the number and check if the sum is a multiple of 3."

Determining multiples of 3

To see if a number is a multiple of 3, add its digits and check if the sum is a multiple of 3. For example, to check if 123,456,789 is a multiple of 3, add

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45.$$

45 is a multiple of 3, so 123,456,789 must be too.

Yuri "You cheated. Somehow."

Me "How is applying a rule cheating?"

Yuri "Well for one thing, how did you add up 1 through 9 so fast?"

Me "I had it memorized."

Yuri "You have *got* to be joking."

Me "Well, actually I memorized that the sum of 1 through 10 is 55, so I just subtracted 10 from that."

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$$

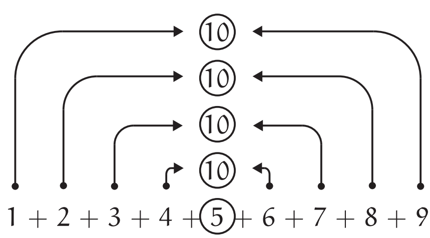
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 \qquad = 45$$

Yuri "Forgive me if I don't waste precious brain cells remembering that."

Me "You don't really need to. It's almost just as easy to sum it up using paired tens."

Yuri "Meaning?"

Me "You can add the first 1 and the last 9 to get 10, right? Then do the same with 2 and 8, 3 and 7, and 4 and 6. That gives you four 10s and a 5 left over. Add those up to get 45."



Paired tens make this addition problem easy

Yuri "Now *that* I like!"

Me "How'd you come up with the problem?"

Yuri "My math teacher showed us how to check if a number is a multiple of 3 by adding its digits and seeing if the sum is a multiple of 3."

Me "So it's cheating when I use a rule, but not when you do?"

Yuri "Glad to see you're catching on."

1.2 Another Check

Me "How about giving this one a shot?"

Problem

Is 103,690,369 a multiple of 3?

Yuri "Piece of cake. Let's see, $1+0+3+6+9+0+3+6+9$ is...uh...37. And 37 isn't a multiple of 3, so 103,690,369 isn't either!"

Answer

103,690,369 is **not** a multiple of 3.

Me "That's right. Took you long enough, though."

Yuri "Well excuse me, Mr. Human Calculator."

Me "Actually, you don't need to calculate anything for this one."

Yuri "Not even paired tens?"

Me "Nope. When you use the rule, you can ignore digits that are already multiples of three."

Yuri "Whaaat?"

Me "You only need to add up the digits that *aren't* multiples of three."

$$1+\underbrace{0+3+6+9+0+3+6+9}_{\text{you can ignore these multiples of 3}}$$

Yuri "That only leaves a 1!"

Me “That’s right. And since 1 isn’t a multiple of 3, this number can’t be a multiple of 3.”

Yuri “More cheating!”

Me “Not at all. Do you see how adding a multiple of 3 to another multiple of 3 just gives you a new multiple of 3?”

Yuri “I guess.”

Me “And how adding a multiple of 3 to a number that *isn’t* a multiple of 3 *won’t* result in a multiple of 3?”

Yuri “Hmm . . . ”

1.3 Mathematical Proof

Me “It sounds like you haven’t convinced yourself that this rule works.”

Yuri “What do you mean?”

Me “That you obviously know how to *use* the rule, but you don’t understand it well enough to really get it.”

Yuri “Yeah, maybe.”

Yuri frowns and twists her hair with a finger.

Me “If you really want to be convinced, I can show you a mathematical proof.”

Yuri “What’s a proof?”

Me “It’s where you apply certain conditions to show that some mathematical statement must logically be true.”

Yuri “Not seeing why I would want to do that.”

Me “Because it gets rid of maybes. It’s a way of showing that you’re absolutely, positively right.”

Yuri "I'm always absolutely, positively right, but maybe these proof things would be handy in convincing others."

Me "I figured you'd see the attraction."

Yuri "So show me how to do them."

Me "Let's start by limiting the discussion to numbers less than 1000."

What we want to prove

Let n be an integer, with $0 \leqslant n < 1000$ (i.e., $n = 0, 1, 2, \ldots, 998, 999$), and let A_n be the sum of the digits in n.

Then the following are true:

① If A_n is a multiple of 3, then n is a multiple of 3.

② If A_n is not a multiple of 3, then n is not a multiple of 3.

Yuri "Well that was easy."

Me "No, no—this isn't the proof. This is what we want to prove."

Yuri "I don't remember wanting to prove anything about all these n's and A_n's and all."

Me "We need those to precisely describe what we're doing. Trust me, if you just say things like 'this number' or 'that number,' everything ends up becoming much more confusing."

Yuri "Why can't we just use an actual number, like 123?"

Me "You can. In fact, a lot of times it's best to start out with a specific example like that."

Yuri "Here goes, then. $1 + 2 + 3 = 6$, and 6 is a multiple of 3. To check if that really worked, you divide 123 by 3, and get ... uh ... 41. Since we got a whole number as the answer, that means 123 is a multiple of 3, too, so the rule worked. Done!"

Me "Not done. All you did is verify that the proposition works for one specific number."

Yuri "And that's wrong how?"

Me "It isn't *wrong* at all. Like I always say, examples are the key to understanding. What you just did shows you know exactly what it is we want to prove."

Yuri "But ... ?"

Me "But an example isn't a proof. We want to be more ... general."

Yuri "And what does that mean?"

Me "Letting $n = 123$ is what's called a specific case, and you showed that this statement ① works for that particular value of n. But you can't do that for every number, 0 through 999."

Yuri "You doubt my abilities, do you?"

Me "Let me rephrase. Sure, you *could* check every number, but it would be an awful lot of work, right?"

Yuri "Yeah, and I'm allergic to lots of work."

Me "Then think of letter variables in math as a way to avoid all that."

Yuri "Convince me."

Me "It's called 'generalization through the introduction of a variable.' For example, we can rewrite this n using a, b, c, like this."

Representing numbers as letters

We can represent an integer n $(0 \leqslant n < 1000)$ using letters a, b, c as follows:

$$n = 100a + 10b + c$$

Here, each of a, b, c is one of $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.

Yuri "You're making things worse."

Me "It's not that bad. Can you rewrite $100a + 10b + c$ using multiplication symbols?"

Yuri "Like this?"

$$100 \times a + 10 \times b + c$$

Me "Right. See how we're adding 100 times a, 10 times b, and c?"

Yuri "Yeah, but what's a?"

Me "It's the digit in the hundreds place of the number. And b is the tens digit, and c is the ones digit."

Yuri crosses her arms and frowns.

1.4 Defining Things for Yourself

Me "Something bothering you?"

Yuri "Yeah. How can you know all that? That the hundreds digit is a and all."

Me "It's not that I know what it is, I just defined things that way. Those are definitions I made to use in the proof I'm going to do."

Yuri "You can just make stuff up like that?"

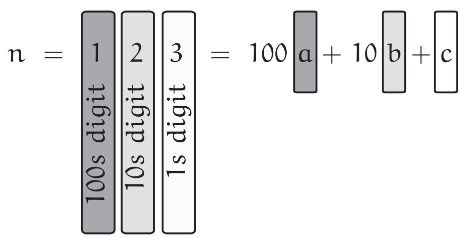
Me “Sure, you can define things any way you like. I guess it seems weird at first, but when you use equations to work problems out, defining your own variables can be a real help. I named these variables a, b, c, but anything would do. Use whatever letters you like.”

Yuri “Go on then.”

Me “Okay, getting back to the problem. We rewrote n as

$$n = 100a + 10b + c.$$

We're using a to represent the number in the hundreds place, b as the number in the tens place, and c as the number in the ones place. So for example, if $n = 123$, then $a = 1$, $b = 2$, and $c = 3$.”



- a is one of $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$
- b is one of $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$
- c is one of $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

Me “So are you comfortable representing n as $100a + 10b + c$?”

Yuri “Got it!”

1.5 From Ideas to Math

Me “This is good practice for representing mathematical objects as statements.”

Yuri “What’s a mathematical object?”

Me “It’s just a fancy name for a mathematical *something*, like an integer that has some value 0 or higher, but less than 1000. When you’re writing a proof, you’ll want to write ideas like that as a mathematical expression. ‘Integer n such that $0 \leqslant n < 1000$,’ for example. Then I took that a step further, and used a, b, c to represent the number as $100a + 10b + c$.”

Mathematical Object		Mathematical Expression
an integer that’s at least 0, but less than 1000	→	$n = 100a + 10b + c$ $0 \leqslant n < 1000$

Yuri “It still feels like you’re just making a simple thing more complex.”

Me “Sure, all these letters will be confusing if you aren’t sure what they represent. But if you take the time to understand what they’re doing, there’s nothing to be afraid of.”

Yuri “Who said anything about being afraid? It just looks like too much work.”

Me “Not if you have a good reason for doing things that way.”

Yuri “Then hurry up and show me what that good reason is. You were going to prove something, right?”

Me "Moving on, then. The next thing I wanted to do was add the digits of the number, and call their sum A_n. Do you see how to do that?"

Yuri "Nothing hard there. You just write $A_n = a + b + c$."

The sum of the digits in n can be written as

$$A_n = a + b + c.$$

Me "Exactly. Since we set things up so that a, b, c will be the digits in the number, all we have to do is add those three variables to find the sum of the digits in n."

Yuri "So get to the part where you explain why we did all that."

Me "Okay, but first let me write a summary of what we've done."

What we have so far

We wrote an integer n (where $0 \leqslant n < 1000$) as

$$n = 100a + 10b + c.$$

We named the sum of n's digits A_n, so

$$A_n = a + b + c.$$

Yuri "No problem there."

Me "And here's what we want to prove."

What we want to prove

Let n be an integer, with $0 \leqslant n < 1000$ (i.e., $n = 0, 1, 2, \ldots, 998, 999$). Let A_n be the sum of the digits in n.

Then the following are true:

① If A_n is a multiple of 3, then n is a multiple of 3.

② If A_n is not a multiple of 3, then n is not a multiple of 3.

Yuri "So prove it already."

1.6 The Power of Mathematical Statements

Me "Things will be easier if we rewrite what we want to prove as mathematical statements."

What we want to prove, rewritten

① If $a + b + c$ is a multiple of 3, then $100a + 10b + c$ is a multiple of 3.

② If $a + b + c$ is not a multiple of 3, then $100a + 10b + c$ is not a multiple of 3.

Yuri "You just replaced the A_n's with $a + b + c$, and the n's with $100a + 10b + c$, right?"

Me "Right. Now that everything is sorted out, let's try yanking out all the multiples of 3 that we can."

Yuri "What do you mean, yank them out?"

Me "I mean doing this."

$100a + 10b + c$	$= 99a + a + 10b + c$	rewrite $100a$ as $99a + a$
	$= 3 \times 33a + a + 10b + c$	rewrite $99a$ as $3 \times 33a$
	$= 3 \times 33a + a + 9b + b + c$	rewrite $10b$ as $9b + b$
	$= 3 \times 33a + a + 3 \times 3b + b + c$	rewrite $9b$ as $3 \times 3b$
	$= 3 \times 33a + 3 \times 3b + a + b + c$	change order of addition
	$= 3 \times (33a + 3b) + a + b + c$	factor out a 3
$100a + 10b + c$	$= 3 \times (33a + 3b) + a + b + c$	the final result

Me "Whatcha think?"

Yuri "What *is* this mess? Why write $100a$ as $3 \times 33a + a$?"

Me "Like I said, I want to yank out all the factors of 3 that I can."

Yuri "But why do you want to do that?"

Me "Just look at the final form of the equation."

$$100a + 10b + c = 3 \times (33a + 3b) + a + b + c$$

Yuri "What am I looking for?"

Me "Uh, maybe it will be easier to see if I change the order of things."

$$100a + 10b + c = a + b + c + 3 \times (33a + 3b)$$

Yuri "Or maybe not."

Me "Do you see how this $3 \times (33a+3b)$ part here is a multiple of 3?"

Yuri "Sure, because it's something that's being multiplied by 3."

Me "Well that means the right side of this equation is $a + b + c$, plus some multiple of 3."

$$100a + 10b + c = a + b + c + \underbrace{3 \times (33a + 3b)}_{\text{a multiple of 3}}$$

Yuri “And?”

Me “And, like we said before, if you add a multiple of 3 to some number, if that number was a multiple of 3 then the result will be another multiple of 3. If it wasn’t, then the result can’t be a multiple of 3. So this equation says that whether or not $100a + 10b + c$ is a multiple of 3 depends on whether $a + b + c$ is a multiple of 3.”

Yuri “Which is what we were aiming for.”

Me “Yep, and that completes the proof. We now know for sure that if you want to see if a number is a multiple of 3, you just have to check if the sum of its digits is a multiple of 3.”

What we proved

Let n be an integer, with $0 \leqslant n < 1000$ (i.e., $n = 0, 1, 2, \ldots, 998, 999$). Let A_n be the sum of the digits in n.

Then the following are true:

① If A_n is a multiple of 3, then n is a multiple of 3.

② If A_n is not a multiple of 3, then n is not a multiple of 3.

Me “We’ve only proved this works for numbers less than 1000, but we can generalize things further. Here’s where the math gets really fun. First, we—”

Yuri “Whoa, hold up there.”

Me “What’s wrong?”

Yuri “I can follow the proof. I see the logic and all, at least. But something still isn’t quite clicking.”

Me “Something in particular?”

Yuri "That thing about adding 3 to a number, and whether the result is a multiple of 3 depending on the first number. That feels, I dunno, kinda weak."

Me "Ah, good point. Okay, let's make it click!"

1.7 Remainders

Me "I think this is what's bothering you, written out precisely."

Yuri's question

Let n be a nonnegative integer ($n = 0, 1, 2, 3, \ldots$). Then,

① If n is a multiple of 3, then the sum of n and a multiple of 3 is a multiple of 3.

② If n is not a multiple of 3, then the sum of n and a multiple of 3 is not a multiple of 3.

Yuri "Yeah, I think that covers it. It makes sense, I guess, but still . . . "

Me "It will make more sense if you think about remainders after dividing by three."

Yuri "How does that help?"

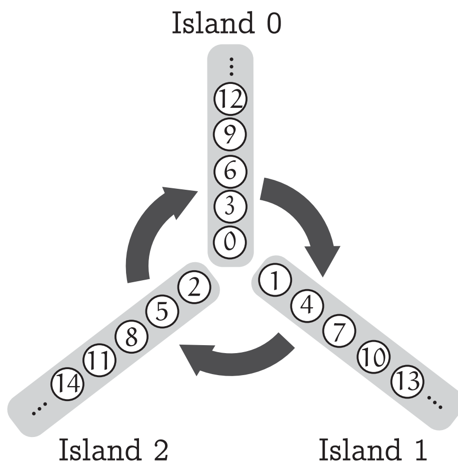
Me "If you divide an integer by 3, there are three possible remainders: 0, 1, or 2."

Yuri "A remainder of 0? That would be no remainder, right? Like, the number was evenly divisible by 3."

Me "Yeah, sure. But the important thing is that there are three possibilities."

Yuri "Okay."

Me "Think about it like this."



Yuri "What on earth is this?"

Me "Imagine we have three islands, Island 0, Island 1, and Island 2. You put the nonnegative integers on those islands according to these rules."

- Integers that leave a remainder of 0 after division by 3 go onto Island 0.
- Integers that leave a remainder of 1 after division by 3 go onto Island 1.
- Integers that leave a remainder of 2 after division by 3 go onto Island 2.

Me "So 0 goes to Island 0, 1 goes to Island 1, and 2 goes to Island 2, right?"

Yuri "Sure, I see that."

Me "So where does 3 go, since there's no Island 3?"

Yuri “To Island 0, of course, since dividing 3 by 3 doesn’t leave a remainder.”

Me “Yep. And 4 goes to Island 1, 5 goes to Island 2, 6 goes to—”

Yuri “Enough already, I get it. They go around and around, right? Every time you add 1 you move on to the next island.”

Me “That’s right, you move around in the direction of the arrows. So what happens when you add 3 to a number?”

Yuri “Um . . . I guess you don’t get anywhere. Because adding 3 means you end up on the same island you started from.”

Me “And that answers your question.”

Yuri “It does? Hey, it does! Because if you started on Island 0, where all the multiples of 3 are, then you’ll just stay there. Same for the other islands, which don’t have any multiples of 3!”

Me “Exactly. And adding a multiple of 3 means you’re just adding 3 a certain number of times, so it’s the same thing.”

Yuri “That clicking sound you just heard was me *totally* getting this. Okay, I’m absolutely convinced.”

Me “Great! Now we’re up to speed on the proof we did for numbers less than 1000. So like I said, the real fun comes when we generalize to—”

Yuri “Whoa, hold up.”

1.8 Yuri's Proposal

Yuri "So about this method for checking if a number is a multiple of 3 . . . "

Me "What about it?"

Yuri "I think I found an easier way, without messing around with all the a's and b's and all that."

Me "This should be interesting."

Yuri "You add all the digits to do the check, right?"

Me "Yeah, sure."

Yuri "Well, 0 is a multiple of 3, so just go adding 1s, and you get multiple, not-a-multiple, not-a-multiple, multiple, not-a-multiple, not-a-multiple, like that."

Me "You lost me."

Yuri "Okay, I'll go slower for you. We know 0 is a multiple of 3, right?"

Me "Yes, it is."

Yuri "And 1 is not a multiple of 3."

Me "Correct."

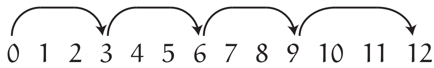
Yuri "And 2 isn't a multiple of 3 either."

Me "It is not."

Yuri "So $0, 1, 2$ goes multiple, not-a-multiple, not-a-multiple, yeah?"

Me "Oh, what you want to say is that we hit a multiple of 3 on every third number?"

Yuri "That's what I *am* saying!"



Every third number is a multiple of 3

Me “Well that’s neat I guess, but we’re talking about ways to check for—”

Yuri “That’s what this is! Pay attention! All the multiples of 3 come every third number. But when you’re adding, everything’s cool up to 9. The only difference is if you’re adding to just the ones digit, or if you’re adding to the tens digit, too. Get it?”

Me “Sorry, Yuri. I’m still lost.”

Yuri “Ugh! Why don’t you get this? Are you being dense on purpose?”

Yuri seems close to tears. I bite my lip and proceed carefully.

1.9 Yuri’s Explanation

Me “Hey, Yuri. Let’s give it one more shot, a little slower this time.”

Yuri glares at me, making me worry that I’ve lost her, but she finally continues.

Yuri “I’m thinking in turn from 0.”

Me “Okay, from 0 then.”

Yuri “It works for 0, right?”

Me “Uh . . . what works?”

Yuri “Gah! If you’re checking numbers to see if they’re multiples of 3 by adding digits and seeing if that gives you a multiple of 3, that works when the number you’re checking is 0, right?”

Me "If $n = 0$ then $A_n = 0$, and both are multiples of 3, so sure, that works."

Yuri "So start from 0 and go up one at a time, through $1, 2, 3, 4, 5, 6, 7, 8, 9$. When you do that, A_n and n both increase by 1 each time, right? So if A_n is a multiple of 3 then n will be too, because $A_n = n$ the whole way."

Me "I see that, yeah."

Yuri "So the only thing you have to worry about is when you move up a digit. You've just got to make sure everything works then."

Me "When you move up a digit?"

Yuri "When you add 1 to a 9 somewhere."

Me "Ah hah! I'm starting to see where you're heading."

Yuri "Then the 9 becomes a 0, and the next unit up gets a 1 added to it, right?"

Me "It does indeed."

Yuri "Well that means the sum of all the digits is having 9 taken out of it, then 1 added!"

Me "Okay, I think I'm following you, but let me make sure. You're saying that if you add 1 to n, and doing that results in two digits being changed, then the effect on the sum of the digits is to subtract 9, then add 1. Like, if $n = 129$ then $A_n = 1 + 2 + 9 = 12$. When you add 1 to n it becomes 130, so A_n becomes $1 + 3 + 0 = 4$. And that 4 is the same thing you get from subtracting 9 from the first A_n, which was 12, then adding 1."

Yuri "Yes! Exactly!"

Me "So as an equation it looks something like this."

$$A_{n+1} = A_n - 9 + 1 \qquad \text{the effect of two digits changing}$$

Yuri "Uh . . . yeah, that's it."

Me "What about when more than two digits change, like when you add 1 to 99 and get 100?"

Yuri "You subtract 9 for each 0 that popped up, then add 1. So you end up subtracting some multiple of 9, and adding 1 at the end, right?"

Me "Wow, Yuri, this is good. So let's see . . . When you add 1 to n, you're subtracting some multiple of 9 from the sum of n's digits, and adding 1. As an equation, that looks like this."

$$A_{n+1} = A_n - 9m + 1 \qquad m \text{ is the number of 0s that appear } (m = 0, 1, 2, \ldots)$$

Yuri "But subtracting a multiple of 9 means you're subtracting a multiple of 3. On the picture you drew, that means you're just spinning backwards around the three islands some number of times before you end up back where you started."

Me "Because 9 is a multiple of 3. Yeah, you're right."

Yuri "So even when more than one digit changes, the effect is the same as if just one digit increased by 1. And *that* means that if $n + 1$ is a multiple of 3, then A_{n+1} is too, and if $n + 1$ isn't, then A_{n+1} isn't either."

Yuri sits back, glowing with self-satisfaction.

Me "This is really cool, Yuri."

Yuri "Isn't it?"

Yuri's discovery

As you increase n in order from $0, 1, 2, \ldots$, either both n and the sum of its digits A_n will be multiples of 3, or they both won't.

Me — "By the way, you can use the same kind of method to check for multiples of 9."

Yuri — "Yeah, we did that one in class, too."

Me — "You did?"

Yuri — "Sure. You're talking about adding the digits in a multiple of 9, and seeing if that gives a multiple of 9, right?"

Me — "Right. And what you discovered here made me realize why that works—because when other digits change it's the same as subtracting 9 and adding 1 to the sum of the digits, just like in this problem."

Yuri — "Feel free to call me when you need more math mysteries solved."

Me — "Hey, wait a second—"

My mom calls from the dining room.

Mom — "You kids want some cookies?"

Yuri — "We're on our way!"

Me — "But I—"

Yuri — "Didn't you hear? *Cookies!*"

I kept thinking as Yuri dragged me down the hall. The methods for checking for multiplicity by 3 and 9 were definitely related to Yuri's discovery, because 3 and 9 both divide 9 evenly. But what's so special about 9?

Of course! It's because we count in decimal, in base-10! So couldn't we generalize that to base-n*? Couldn't we create a rule for determining if numbers in base-*n *were multiples of* $n-1$*, using this "subtracting* $n-1$ *and adding* 1*" pattern?*

"...but if you don't understand how rules work, you'll never be able to improve them."

Problems for Chapter 1

Problem 1-1 (Checking for multiples of 3)

Which of (a), (b), and (c) are multiples of 3?

(a) 123456

(b) 199991

(c) 111111

(Answer on page 187)

Problem 1-2 (Representation as mathematical statements)

Let n be an **even** integer in the range $0 \leqslant n < 1000$. Letting a, b, c respectively be the hundreds, tens, and ones digits of n, what values can a, b, c take?

(Answer on page 188)

Problem 1-3 (Building a table)

In this chapter, the narrator used A_n to represent the sum of the digits in an integer n. For example, when $n = 316$,

$$A_{316} = 3 + 1 + 6 = 10.$$

Using that, fill in the blanks in the following table:

n	0	1	2	3	4	5	6	7	8	9
A_n										

n	10	11	12	13	14	15	16	17	18	19
A_n										

n	20	21	22	23	24	25	26	27	28	29
A_n										

n	30	31	32	33	34	35	36	37	38	39
A_n										

n	40	41	42	43	44	45	46	47	48	49
A_n										

n	50	51	52	53	54	55	56	57	58	59
A_n										

n	60	61	62	63	64	65	66	67	68	69
A_n										

n	70	71	72	73	74	75	76	77	78	79
A_n										

n	80	81	82	83	84	85	86	87	88	89
A_n										

n	90	91	92	93	94	95	96	97	98	99
A_n										

n	100	101	102	103	104	105	106	107	108	109
A_n										

(Answer on page 189)

About this sample chapter

Thank you for reading this sample. I hope you enjoyed reading it as much as I enjoyed translating it.

This is the second volume in a series of books that aims at addressing select topics from a variety of areas of mathematics. These books present fun excursions that learners are less likely to see in typical classroom settings. They attempt to illustrate problem solving techniques in those areas, and to show how fun and exciting mathematics can be (especially when enjoyed with friends!). The presentation is via dialogue between characters from the *Math Girls series* of books, which generally contain higher level math.

The first volume, *Math Girls Talk About Equations & Graphs*, develops topics such as using variables in equations, polynomials, setting up systems of equations, proportions and inverse proportions, the relation between equations and their graphs, parabolas, intersections, and tangent lines.

In addition to what you just read, *Math Girls Talk About Integers* covers topics such as finding primes, the interesting mathematics behind a magic trick, and using remainders from division to solve a problem based on clocks. A "final boss" chapter tackles a challenging proof by mathematical deduction, a problem from an actual Japanese college entrance examination.

The next volume, *Math Girls Talk About Trigonometry*, is scheduled for release around December 2014. That book will of course take up topics from trigonometry, from the basics of how the trigonometric functions are defined to fun explorations of Lissajous curves, methods of estimating pi, and using vectors to find the trigonometric addition formulas.

More books are planned, covering topics such as sequences and series, the mathematics of change, math with vectors, and probability. We hope you enjoy them all.

If you would like to purchase the full version of this book, you can order it directly from our printer, via major online booksellers, or by special order at your local book store. We hope you enjoy it, and the rest of the *Math Girls* books!

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Index

Other works by Hiroshi Yuki

(in English)

- *Math Girls*, Bento Books, 2011
- *Math Girls 2: Fermat's Last Theorem*, Bento Books, 2012
- *Math Girls Manga*, Bento Books, 2013
- *Math Girls Talk About Equations & Graphs*, Bento Books, 2014

(in Japanese)

- *The Essence of C Programming*, Softbank, 1993 (revised 1996)
- *C Programming Lessons, Introduction*, Softbank, 1994 (Second edition, 1998)
- *C Programming Lessons, Grammar*, Softbank, 1995
- *An Introduction to CGI with Perl, Basics*, Softbank Publishing, 1998
- *An Introduction to CGI with Perl, Applications*, Softbank Publishing, 1998

- *Java Programming Lessons (Vols. I & II)*, Softbank Publishing, 1999 (revised 2003)
- *Perl Programming Lessons, Basics*, Softbank Publishing, 2001
- *Learning Design Patterns with Java*, Softbank Publishing, 2001 (revised and expanded, 2004)
- *Learning Design Patterns with Java, Multithreading Edition*, Softbank Publishing, 2002
- *Hiroshi Yuki's Perl Quizzes*, Softbank Publishing, 2002
- *Introduction to Cryptography Technology*, Softbank Publishing, 2003
- *Hiroshi Yuki's Introduction to Wikis*, Impress, 2004
- *Math for Programmers*, Softbank Publishing, 2005
- *Java Programming Lessons, Revised and Expanded (Vols. I & II)*, Softbank Creative, 2005
- *Learning Design Patterns with Java, Multithreading Edition, Revised Second Edition*, Softbank Creative, 2006
- *Revised C Programming Lessons, Introduction*, Softbank Creative, 2006
- *Revised C Programming Lessons, Grammar*, Softbank Creative, 2006
- *Revised Perl Programming Lessons, Basics*, Softbank Creative, 2006
- *Introduction to Refactoring with Java*, Softbank Creative, 2007
- *Math Girls / Fermat's Last Theorem*, Softbank Creative, 2008

- *Revised Introduction to Cryptography Technology*, Softbank Creative, 2008
- *Math Girls Comic (Vols. I & II)*, Media Factory, 2009
- *Math Girls / Gödel's Incompleteness Theorems*, Softbank Creative, 2009
- *Math Girls / Randomized Algorithms*, Softbank Creative, 2011
- *Math Girls / Galois Theory*, Softbank Creative, 2012
- *Java Programming Lessons, Third Edition (Vols. I & II)*, Softbank Creative, 2012
- *Etiquette in Writing Mathematical Statements: Fundamentals*, Chikuma Shobo, 2013
- *Math Girls Secret Notebook / Equations & Graphs*, Softbank Creative, 2013
- *Math Girls Secret Notebook / Let's Play with the Integers*, Softbank Creative, 2013
- *The Birth of Math Girls,* Softbank Creative, 2013
- *Math Girls Secret Notebook / Round Trigonometric Functions*, Softbank Creative, 2014